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THERMAL PROPERTIES AND TEMPERATURE DISTRIBUTION OF SNOW / FIRN ON THE LAW DOME ICE CAP, ANTARCTICA

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Abstract Based on detailed measurements of density and a numerous data on temperature in shallow boreholes (about 20m deep), the thermal properties and temperature distribution of snow / firn layer on the Law Dome ice cap, Antarctica, are discussed. According to a review of works on thermal properties of snow by Yen (1981), a relationship between thermal conductivity (K) and density (ρ) is proposed to be expressed by a formula, $K=0.0784+2.697\rho^2$. Then an eqation of heat transfer in a deformed ununiform medium is applied and solved analytically by two approaches. Comparison of calculated and measured temperatures indicates that the difference is mainly dependent on the determination of boundary donditions.

Key words Law Dome Ice Cap, snow-firn layer, thermal properties, temperature distribution.

1. Introduction

The temperature regime of the near-surface ice-firn layer or the active layer, in which temperature changes with the seasonal variation of climate, on glaciers and ice sheets has been discussed in many papers and some books, such as in Zotikov (1986), Paterson (1981) and Budd (1969). However, the thermal parameters were mostly regarded as constants in a glacier. This is reasonably approximate in the ablation area, where the snow / firn is generally thiner. In the case of thicker snow / firn layer, in the accumulation area, the thermal conductivity and diffusivity, vary in a large range because of much more variation of snow /firn density than ice. Although the thermal parameters are related to other factors, such as temperature and impurity content in snow and ice, the density effect has been proven to be the most important by numerous researches on snow thermal properties in laboratory and field measurements (Yen, 1981).

In general the density of snow firn on a glacier increases with depth, but may be quite different from place to place. Therefore, it is difficult to discuss meticulously the thermal properties and temperature distribution of the active layer within a glacier without the detailed density measurements. On the other hand, in many papers concerning the glacier temperature, only a temperature profile across the whole glacier or ice sheet is given ; so the thermal parameters are usually considered to be constant as the snow / firn layer is only a tiny fraction. For example, Budd and others (1976) have made an excellent work on the temperature regime of the Law Dome ice cap, Antarctica, from a steadystate and uniform medium assumption. The Law Dome ice cap has been investigated for 30 years. Cameron and Bull (1962) measured of annual-cyclic variation of temperature at depth of 16m in the ablation area at 262m a.s. l. and calculated inversively the mean thermal parameters through a solution of heat conductivity equation of a semi-infinite uniform medium with a harmonic ally changing boundary condition. Recent years the similar work has been done in four boreholes at 20m depth (Figure 1) by some Chinese glaciologists who joined in ANARE (the Australian National Antarctic Research Expedition) (Qin and others, 1988). Snow / firn density was measured minutely in the boreholes in the accumulation area (LJ, LJ24 and BJ in Figure 1). In the present paper an attempt to determine the thermal parameters of snow / firn layer is made from the obtained data and a previously empirical relationship between thermal conductivity and density of snow proposed by some investigators after their laboratorial experiments and field measurements, and then to discuss the model of temperature distribution in the near-surface snow / firn layer treated as an ununiform medium, with the emphasis on Borehole BJ, which is at highest elevation (1043m a.s.l.) , where the effect of meltwater on the surface is relatively slight.

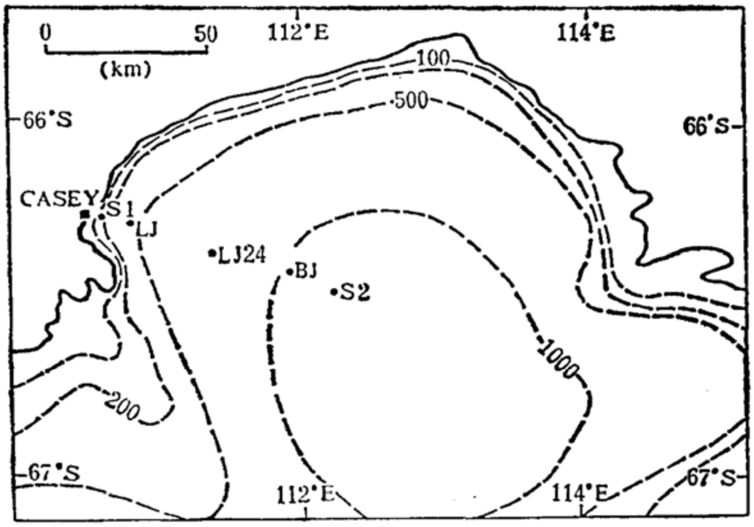

Fig. 1. Map of the Law Dome showing the locations of boreholes for temperature measurement.

2. Measurements of Density and Temperature

The Borehole BJ, is located at 66°28'S, 111°58'E and 1043m a.s. l., 80km from Casey Station. At this site the present accumulation rate is 77kg / m^2a, as determined from stake measurements, and the surface melting is so faint that each layer of ice formed in snow / firn is ony 1 to 2mm thick. Xie (1984) considered the site in the cold infiltration-recrystallization zone to be equivalent to the upper part of the percolation zone in Paterson's (1981) diagram. The hole was drilled by means of a light-type PICO auger down to 21.34 m depth. Seven platinum thermometers were buried in it at depths of 1,2.5, 5, 7.5, 10, 15 and 20m respectively. The temperature measurement started in May, 1984, and lasted two years and longer with an interval of about a half month in summer or one month in the other seasons. The details of installation and calibration of the thermometers were described by Qin and others (1988), who believed the accuracy of temperature data to be 0.01K. The data used in the present paper are collected by late January, 1986, and listed in Table 1.

After the hole was drilled, the cores were measured and analyzed for mutiple purposes, including density variations with depth. The density measurement was made as follows: the cores were cut into disks with a thickness of about 20mm, and then a vernier caliper with its resolution of 0.002 mm was used to measure each dimension of the samples. Finally, the mass measurement was made on a balance with its resolution of 0.01 g. The relative error of resulting density value is estimated within one percent (Qin and others, 1988). Figure 2 gives a density profile measured at each 0.02 m interval in Borehole BJ. We can see in this figure that the density points are quite scattered in the upper 10 m depth.

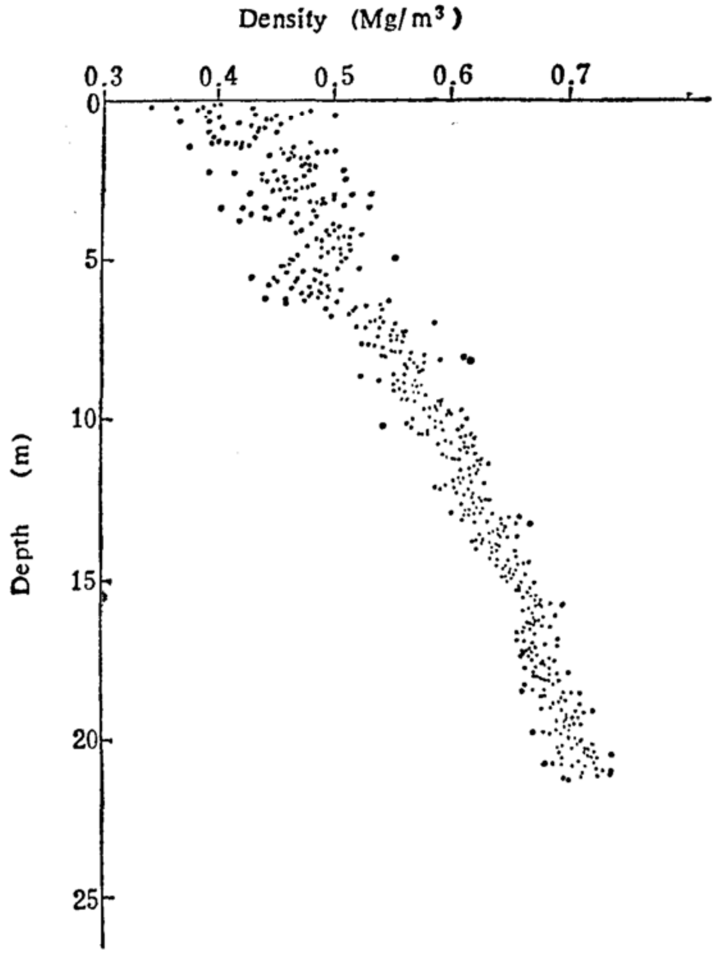

Fig. 2. Snow / firn density profile of Borehole BJ.

3. Determination of Thermal Parameters

As mentioned above, many investigators have reported their results of experimental researches and field measurements on snow thermal conductivity and empirically correlated the conductivity with snow density. Yen (1981) summarized the reported data and suggested an expression

$$\lambda = 2.22362\rho^{1.885}, \tag{1}$$

where λ is effective thermal conductivity in W / (m•K) and ρ is density in Mg / m³. Because of much more complicated heat transfer process in snowpack than in ice, including convection, radiation and vapor diffusion besides conduction, thermal conductivity is expressed as effective thermal conductivity to account for the whole process.

The density measurement shows that the snow / firn density in Borehole BJ on Law Dome is mainly between 0.4 and 0.7 Mg / m³ (Figure 2). For this range, Formula (1) gives generally lower values of λ than experimental or field measured ones (Figure 3). Additionally Yen (1965) obtained a quadratic equation for ρ between 0.5 and 0.59 Mg / m³ :

$$\lambda = 3.2217\rho^2, \tag{2}$$

and the majority of other investigators also gave similar results (cf. Yen, 1981). Therefore, we supposed that a quadratic equation is valid for Borehole BJ on Law Dome and got a fitted curve from the points for $\rho \geqslant 0.4$ in Figure 3:

$$\lambda = 0.0784 + 2.697\rho^2. \tag{3}$$

It gives a value of 2.346 W / m•K for pure ice ($\rho = 0.91 \mathrm{Mg} / \mathrm{m}^3$), as extrapolated, slightly higher than the commonly adopted value of 2.1 to 2.2 W / m•K (see Paterson, 1981 and Yen, 1981).

The effective diffusivity of snow / firn, k, is defined as

$$k = \lambda / (\rho c), \tag{4}$$

where c is specific heat capacity of snow / firn, which is independent of density and can be approaximately regarded as a constant. Although extensive studies have shown the dependence of specific heat capacity of snow / firn and ice on temperature condition (see Yen, 1981), it is reasonably approximate that the specific heat capacity is taken as a constant for a specified site on a glacier because temperature range at it is usually not very large. For Borehole BJ on Law Dome, the difference of measured temperatures below 1 m over two years is less than 12K (see Table 1), which would result in a difference of about 4% at most in calculation of specific heat from the estimate

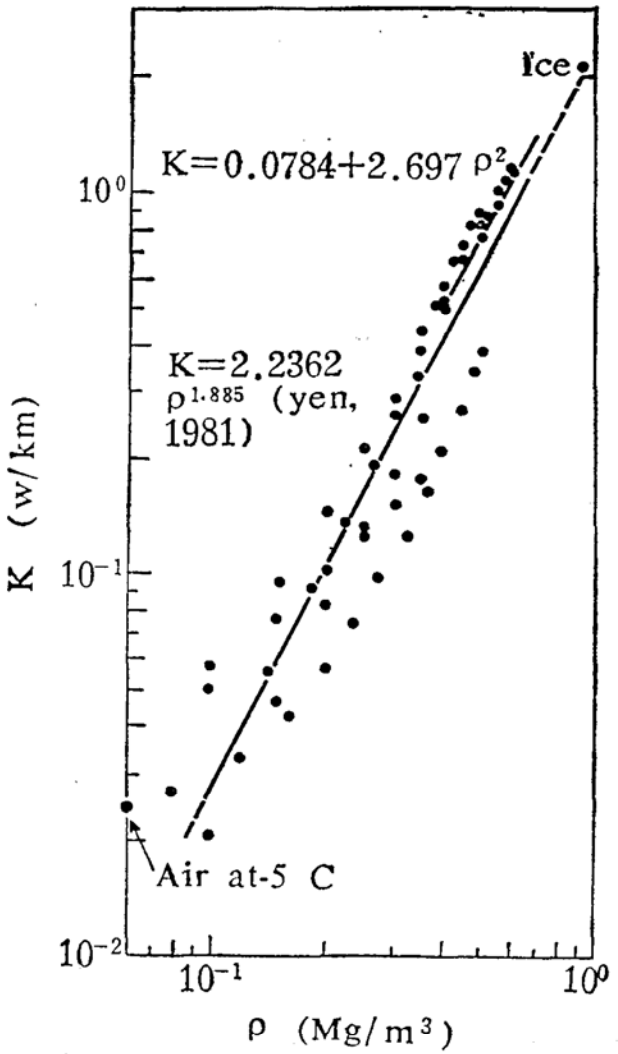

Fig. 3. Effective thermal conductivity of snow as a function of density (after Yen, 1981).

by Yen (1981).

4. Heat Transfer Equation

Considering the ice cap as a deforming continuous medium, the general equation for heat transfer in it can be derived from the law of conservation of energy as follows (Paterson and Clarke, 1978):

$$\frac{1}{k}\left(\frac{\partial T}{\partial t}+\mathrm{v}_i\,\frac{\partial T}{\partial x_i}\right)=\frac{f}{\lambda J}+\frac{1}{\lambda}\,\frac{\partial K}{\partial x_i}\,\frac{\partial T}{\partial x_i}+\nabla^2 T \tag{5}$$

where T is temperature, t time, V_i the velocity vector along the coordinate axis x_i, λ thermal conductivity, k thermal diffusivity, J the mechanical equivalent of heat, and f the rate of internal heat production per unit volume, resulting mainly from deformation and heat released by freezing of water in the medium.

For the active layer, certain simplifications can be made:

(1) Because horizontal temperature gradient is much smaller than the vertical gradient, $\frac{\partial^2 T}{\partial x^2}$ and $\frac{\partial^2 T}{\partial z^2}$ can be neglected and $\nabla^2 T=\frac{\partial^2 T}{\partial y^2}$ when we take the origin at the surface of the ice cap, the x-axis horizontal, the y-axis vertical, positive downwards, and the z-axis so as to make the system right-handed.

(2) Borehole BJ is near the margin of the central area of Law Dome, where the surface slope is not large, so the velocity of horizontal movement is also low . Thus the term $u\frac{\partial T}{\partial x}$ and $w\,\frac{\partial T}{\partial z}$ is negligible.

(2) The internal heat is small because the deformation heating is negligible near the surface and the meltwater on the surface is quite little around Borehole BJ as mentioned previously.

(4) Thermal parameters are assumed to be changed only in vertical direction so that $\frac{\partial K}{\partial x}=\frac{\partial K}{\partial z}=o.$

Equation (5) then becomes

$$\frac{\partial T}{\partial t}=k\frac{\partial^2 T}{\partial y^2}+\left(\frac{1}{\rho c}\frac{\mathrm{d}\lambda}{\mathrm{d}y}-\mathrm{v}\right)\frac{\partial T}{\partial y}. \tag{6}$$

Since the active layer is very thin compared with the whole thickness of ice cap, v can be regarded as a constant and must be equal to the value of net mass balance, if we assume that the ice thickness is unchanged. To obtain an analytical solution, the term $\frac{1}{\rho c}\,\frac{\mathrm{d}\lambda}{\mathrm{d}y}$ should be expressed as a simple form. Of course, equation (6) can be solved numerically, but the analytical solution has the advantage of simplicity and clearness for practical appliance. In the following we will make an attempt to simplify equation (6) further to do so.

5. Two Approaches

1. *Laminar Structure Model*

As thermal conductivity varies only in the vertical direction, we can divide the firn into a ser-

Table 1. Measured temperature values in Borehole BJ on Law Dome(°C).

Date	Depth(m)						
	1.0	2.5	5.0	7.5	10.0	15.0	20.0
24/05/84	−16.81	−16.65	−16.68	−17.29	−17.62	−17.72	−17.56
17/06/84	−20.43	−17.39	−16.68	−17.04	−17.49	−17.70	−17.56
22/07/84	−19.27	−18.30	−17.27	−17.17	−17.42	−17.67	−17.57
16/08/84	−20.74	−19.60	−17.77	−17.29	−17.39	−17.64	−17.56
09/09/84	−18.79	−19.12	−18.08	−17.44	−17.42	−17.62	−17.56
26/09/84	−23.09	−20.01	−19.53	−17.62	−17.44	−17.59	−17.56
12/10/84	−19.84	−19.96	−18.31	−17.65	−17.50	−17.59	−17.56
27/10/84	−20.97	−19.80	−18.58	−17.77	−17.52	−17.56	−17.55
09/11/84	−19.96	−19.70	−18.43	−17.77	−17.56	−17.56	−17.55
23/11/84	−18.89	−19.45	−18.58	−17.87	−17.60	−17.56	−17.55
13/12/84	−15.47	−17.36	−18.48	−17.95	−17.67	−17.56	−17.55
28/12/84	−15.19	−17.55	−18.20	−17.87	−17.70	−17.56	−17.55
14/01/85	−13.00	−13.81	−17.98	−17.95	−17.74	−17.59	−17.55
23/01/85	−14.58	−16.81	−17.77	−17.80	−17.72	−17.59	−17.52
28/02/85	−13.78	−14.37	−16.49	−17.52	−17.75	−17.59	−17.52
28/03/85	−13.97	−15.02	−16.49	−17.40	−17.72	−17.59	−17.52
30/04/85	−16.53	−16.30	−16.55	−17.19	−17.54	−17.62	−17.52
26/05/85	−21.39	−18.07	−16.89	−17.16	−17.50	−17.62	−17.52
04/07/85	−23.08	−20.92	−18.58	−17.55	−17.42	−17.57	−17.52
29/08/85	————	−19.99	−18.75	−17.85	−17.58	−17.55	−17.52
27/09/85	−21.10	−20.17	−18.89	−18.02	−17.65	−17.55	−17.52
23/10/85	−21.05	−20.97	−19.38	−18.33	−17.80	−17.55	−17.50
25/11/85	−17.92	−19.05	−19.27	−18.43	−17.90	−17.57	−17.50
26/12/85	−15.01	−17.70	−18.89	−18.49	−18.00	−17.59	−17.50
24/01/86	−16.50	−17.47	−18.25	−18.00	−17.65	−17.65	−17.50

es of thin laminae so that thermal conductivity is approximately constant within each lamina. Then the problem is one of heat conduction in multi-layers of uniform medium. Accumulation rate at the site of Borehole BJ is about 0.1 m / a; thus if we ignore the vertical movement velocity, the equaltion of heat conduction can be reduced to

$$\frac{\partial T_n}{\partial t}=k_n\frac{\partial^2 T_n}{\partial y^2}. \qquad (n-1)l<y<nl,\ t>0, \tag{7}$$

with a interface condition,

$$T_n\ ((n-1)\ l,\ t)\ =\ T_{n-1}\ ((n-1)l,\ t), \tag{8}$$

where n is a number of laminae counted downwards and l the thickness of each lamina.

As usual, we assume that the boundary conditions are

$$T\ (0,\ t)\ =\ T_0(0)\ +A\sin(\omega t)\ \text{for}\ y\ =\ O \tag{9}$$

and

$$T\,(h,\,t) = T_0(h) \qquad \text{for } y = h, \tag{10}$$

where h is the thickness of the active layer, T_0 the equilibrium temperature, A the amplitude, and $\omega/2\pi$ the frequency of the surface temperature change.

Then the solution is given by

$$T\,(y,\,t) = T_0(y) + A\exp\left\{-lp - [y-(n-1)l]\left(\frac{\omega}{2k_n}\right)^{1/2}\right\}\, q\sin\left[\omega t - y\left(\frac{\omega}{2k_n}\right)^{1/2}\right], \tag{11}$$

where p and q are defined as

$$p = \left(\frac{\omega}{2k_1}\right)^{1/2} + \left(\frac{\omega}{2k_2}\right)^{1/2} + \cdots + \left(\frac{\omega}{2k_{n-1}}\right)^{1/2}, \tag{12}$$

and

$$q = \frac{\sin\left[\omega t - l\left(\frac{\omega}{2k_1}\right)^{1/2}\right]\sin\left[\omega t - 2l\left(\frac{\omega}{2k_2}\right)^{1/2}\right]\cdots\sin\left[\omega t-(n-1)\,l\left(\frac{\omega}{2k_{n-1}}\right)^{1/2}\right]}{\sin\left[\omega t - l\left(\frac{\omega}{2k_2}\right)^{1/2}\right]\sin\left[\omega t - 2l\left(\frac{\omega}{2k_3}\right)^{1/2}\right]\cdots\sin\left[\omega t-(n-)\,l\left(\frac{\omega}{2k_n}\right)^{1/2}\right]}. \tag{13}$$

2. *Vertical Movement Model*

If we average k, ρ and $\mathrm{d}K/\mathrm{d}y$ over the thickness of the active layer and substitute them into equation (6), we can obtain a new equation,

$$\frac{\partial T}{\partial t} = k\,\frac{\partial^2 T}{\partial y^2} + S\frac{\partial T}{\partial y}, \tag{14}$$

where k and S are all constants and S is given *by*

$$S = \frac{1}{\rho c}\,\frac{d\lambda}{dy} - v. \tag{15}$$

Equation (14) is equivalent to that of heat conduction of a uniform medium with vertical movement at a velocity S. Under the boundary conditions given by formulas (9) and (10), the solution can be made after Carslaw and Jaeger (1959) and written as follows:

$$T(y,t) = T_0(y) + A\exp\left(\frac{Sy}{2k} - ya^{1/2}\cos\frac{\phi}{2}\right)\sin\left(\omega t - ya^{1/2}\sin\frac{\phi}{2}\right), \tag{16}$$

where a and ϕ are defined as

$$\left(\frac{S^2}{4k^2} + \frac{i\omega}{k}\right) = ae^{i\phi}. \tag{17}$$

Budd (1966) pointed out from magnitude analysis that Formula (16) can be reduced to

$$T\,(y,\,t) = T_0(y) + A\exp\left\{-y\left[\left(\frac{\omega}{2k}\right)^{1/2} - \frac{S}{2k}\right]\right\}\sin\left[\omega t - y\left(\frac{\omega}{2k}\right)^{1/2}\right] \tag{18}$$

with a relative error of less than 1% when S is not larger than 2m / a. This is suitable to Borehole BJ on Law Dome.

6. Results of Temperature Calculation and Discussion

From the researches on temperature regime of the active layer on glaciers in China(Huang and others, 1982; Ren and others, 1985), we assumed that $T_0\,(y) = T_0(O) + by$, where b is a constant, about 0.009 deg. per metre from measured mean temperatures at 10 and 20m depths. According to measured temperatures of air above the surface and snow / firn at a depth of 1 m, the amplitude, A, should be between 5.5 and 16 deg.; thus we took it as 10 deg. roughly. The date on which the highest air temperature occurs was set to the end of January, responding to $\omega t=\pi/2$, though it is not exactly same in different years.

For the laminar structure model, l was taken as 0.5m, over whick k was averaged. For the vertical movement model, k, averaged over the active layer, was calculated to be 0.798×10^{-6}m / s, $\lambda=0.8777$ W / (m•K), $\frac{dk}{dy}=0.0493$W/m^{2}K and $\rho=0.539$ Mg / m^3. The specific heat capacity, c, was taken as 2009J / (g•K) from Paterson (1981).

Table 2 gives some results of temperature calculation at depths of 1, 5 and 10m and the correspondingly measured values. In Table 2 we can see that the calculated temperatures by means of the two approaches are quite close, though the value by the laminar model is relatively close to that measured. Figure 4 gives a comparison between calculated (by the laminar model) and measured values at a depth of one metre and shows that the amplitude of calculated temperatures is generally larger than that of measured ones and the extreme values of them are out of phase. Moreover, some irregular change arises among the measured temperatures. For example, some transient drop may take place during a period of rising temperature. These implies that the change of air temperature is so complicated that the assumption of a harmonic change for the surface conditions is not sure enough and the amplitude at the surface was overestimated. However, a further detailed observation on air temperature is needed to make a better description of the surface conditions.

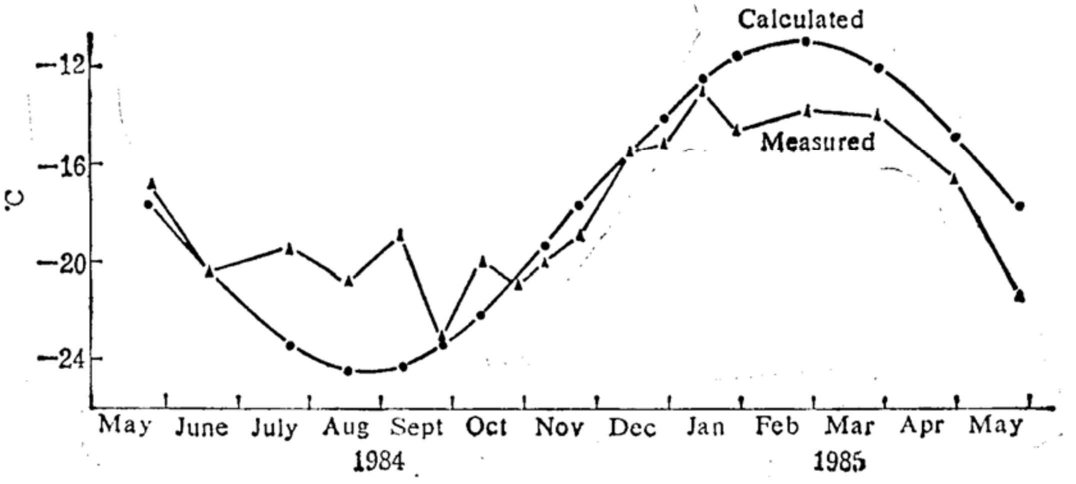

Fig. 4. Comparison between calculated and measured temperatures at a depth of one metre.

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Table 2. A Part of calculated and measured temp. values in Borehole BJ on Law Dome(°C).

Date	1. 0m			5. 0m			10. 0m		
	measured	calculated by		measured	calculated by		measured	calculated by	
		formula(11)	(18)		formula(11)	(18)		formula(11)	(18)
24/05/84	−16. 81	−17. 65	−17. 86	−16. 68	−16. 04	−17. 72	−17. 62	−17. 61	−17. 64
17/06/84	−20. 43	−20. 33	−20. 75	−16. 68	−16. 34	−16. 03	−17. 49	−17. 50	−17. 48
22/07/84	−19. 27	−23. 39	−23. 94	−17. 27	−17. 19	−16. 94	−17. 42	−17. 41	−17. 30
16/08/84	−20. 74	−24. 37	−24. 90	−17. 77	−17. 67	−17. 78	−17. 39	−17. 39	−17. 24
26/09/84	−23. 09	−23. 33	−23. 67	−19. 53	−18. 65	−19. 03	−17. 44	−17. 45	−17. 30
27/10/84	−20. 97	−20. 68	−20. 76	−18. 58	−19. 10	−19. 57	−17. 52	−17. 53	−17. 45
23/11/84	−18. 89	−17. 75	−17. 50	−18. 58	−19. 16	−19. 62	−17. 60	−17. 66	−17. 63
28/12/84	−15. 19	−14. 18	−13. 44	−18. 20	−18. 78	−19. 06	−17. 70	−17. 80	−17. 85
28/01/85	−14. 58	−11. 61	−11. 06	−17. 77	−18. 11	−18. 15	−17. 72	−17. 87	−17. 98
28/02/85	−13. 78	−10. 99	−10. 54	−16. 58	−17. 32	−17. 11	−17. 75	−17. 88	−18. 02
28/03/85	−13. 97	−12. 04	−11. 78	−16. 49	−16. 67	−16. 29	−17. 72	−17. 82	−17. 96
30/04/85	−16. 53	−14. 90	−14. 95	−16. 55	−16. 31	−15. 74	−17. 54	−17. 70	−17. 79

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